

Lec 7:

09/14/2010

Nuclear Energy Production in Stars (Cont'd):

Because of the properties of thermonuclear reaction rates, different burnings are well separated by appreciable temperature differences.

During a certain phase only very few reactions occur with non-negligible rates. The important properties are: astrophysical <sup>correction factors,</sup> "S" factor,  $\Lambda$  and energy release " $Q$ ". These can be found in the literature.

A whole network of all simultaneously occurring reactions must be calculated. Often enough, only the rate for the slowest of a chain of subsequent reaction suffices to be calculated, since it determines essentially the rate of the whole fusion process.

However, the  $Q$  value contains all of the energy made available to the stellar matter by the process. Lets consider two of the major burning phases in stars.

## Hydrogen Burning:

The net result of Hydrogen burning is the fusion of four  ${}^1\text{H}$  into one  ${}^4\text{He}$  and release of 26.43 MeV of energy in the process (which amounts to a mass defect of 0.71%). This is roughly 10 times the energy liberated in any other fusion process.

There are two main series of reactions known as proton-proton chain (pp-chain) and Carbon-Nitrogen-Oxygen cycle (CNO-cycle).

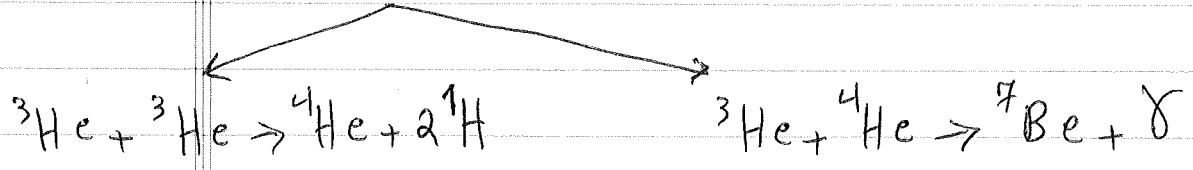
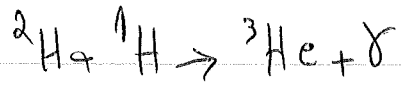
The pp-chain starts with  ${}^2\text{H}$  formation and then  ${}^3\text{He}$  formation:



The first reaction includes  $\beta^+$  decay of a proton, which has a very small cross-section since the decay is governed by weak interactions.

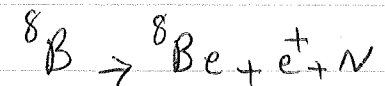
Once  ${}^3\text{He}$  is formed, the completion of  ${}^4\text{He}$  can proceed via

one of three alternative branches, pp1, pp2, and pp3. The reactions involved are shown below;



(pp1)

$$Q = 26.20 \text{ MeV}$$



(pp2)

$$Q = 25.67 \text{ MeV}$$



(pp3)

$$Q = 19.20 \text{ MeV}$$

The  ${}^3\text{He} - {}^4\text{He}$  reaction has a 14% larger reduced mass, a 4.6% larger  $\gamma$  (see equation ~~\*\*\*~~ from previous lecture), and hence a slightly larger temperature sensitivity.

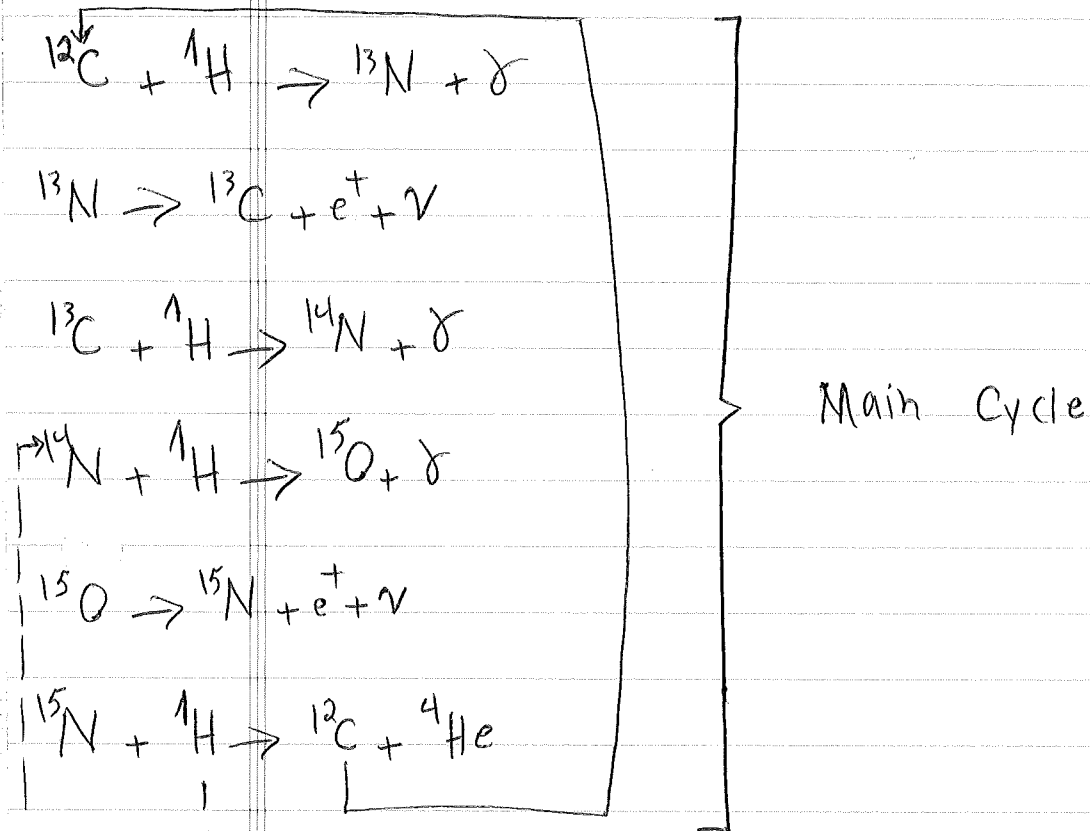
With increasing  $T$ ,  $\rho\rho^2$  and  $\rho\rho^3$  dominate more and more over  $\rho\rho^1$  (say above  $\approx 10^7$  K) if  ${}^4\text{He}$  is present with appreciable amounts. The relative importance will gradually shift toward  $\rho\rho^3$ .

Initially  $\rho\rho^1$  is the main source of energy generation, say for  $T \approx 8 \times 10^6$  K. For gradually increasing  $T$ , and production of  ${}^4\text{He}$ , the energy generation increases by a factor of  $\approx 2$  at  $T \approx 2 \times 10^7$  K. The  $\rho\rho^2$  then takes over as each  ${}^1\text{H}-{}^1\text{H}$  reaction yields one  ${}^4\text{He}$  (compared to every second such reaction in the  $\rho\rho^1$  branch). The energy generation then decreases from  $\approx 2$  to  $\approx 1.5$  where  $\rho\rho^3$  has taken over (note that  $\rho\rho^3$  has a much smaller  $Q$ ).

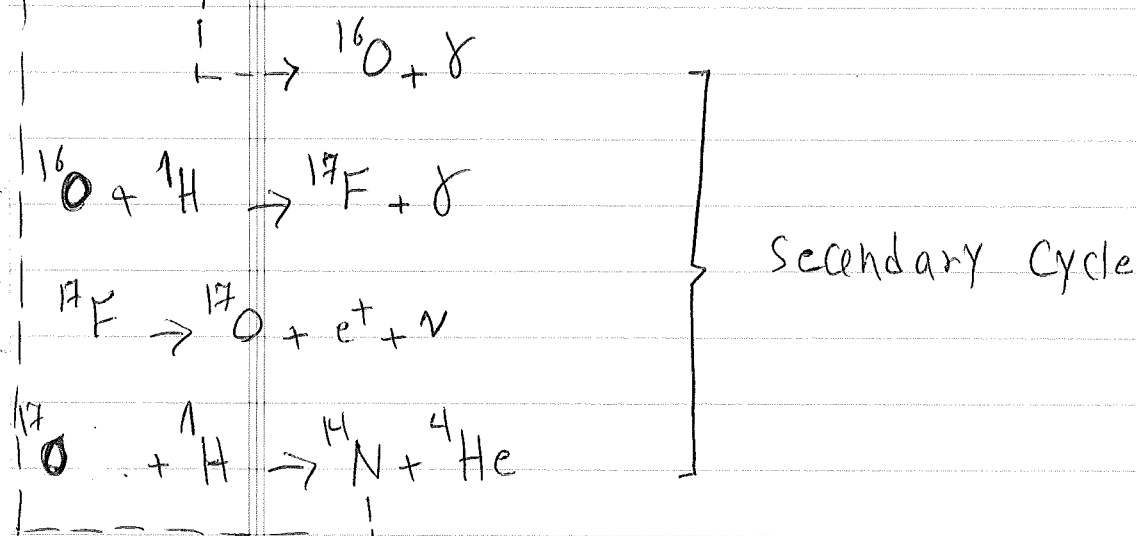
The temperature sensitivity of the  $\rho\rho$ -chain is the smallest of all fusions;  $n \approx 6$  at  $T \approx 5 \times 10^6$  K, which decreases to  $n \approx 3.5$  at  $T \approx 2 \times 10^7$  K.

Next we consider the CNO-cycle. It requires the presence of some isotopes of C, N, or O (as happens in Pop I stars).

The sequence of reactions can be represented as follows:



$$Q = 24.97 \text{ MeV}$$



The secondary cycle is  $\sim 10^4$  times less probable than the main cycle. Most stars change slowly enough that for sufficiently high temperatures (say  $T \gtrsim 1.5 \times 10^7$  K) all the nuclei in the cycle reach their equilibrium abundance. Then it is sufficient to calculate only the slowest reaction, which is  ${}^4\text{N} + {}^1\text{H}$  that essentially controls the cycle. This slowest reaction acts like a bottleneck where the nuclei involved are congested in their flow through the cycle.

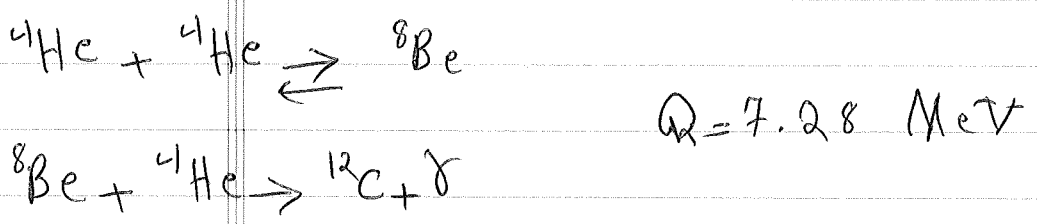
The temperature sensitivity  $\nu$  is much higher than that in the  $\beta\beta$ -chain. For  $T \approx 10^7$  K ( $5 \times 10^7$  K) we have  $\nu \approx 23$  (13). In consequence, the  $\beta\beta$ -chain dominates at low temperatures ( $T < 1.5 \times 10^7$  K), while it can be neglected at higher temperatures.

### Helium Burning:

The reactions consist of the gradual fusion of several  ${}^4\text{He}$  into

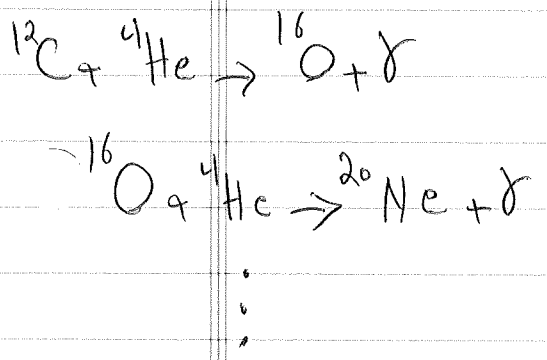
$^{12}\text{C}$ ,  $^{16}\text{O}$ , etc. This requires temperatures  $\gtrsim 10^8\text{ K}$  (because of the larger Coulomb barrier).

The first and key reaction is the triple  $\alpha$  reaction, which is performed in two steps:



The  ${}^8\text{Be}$  decays back into two  $\alpha$  particles with a lifetime of few times  $10^{-16}\text{ s}$ . This is  $\sim 10^5$  larger than the duration of a normal scattering process, hence  ${}^8\text{Be}$  can participate in the second reaction in above.

Once a sufficient  $^{12}\text{C}$  abundance has been built, further  $\alpha$  capture can occur to form  $^{16}\text{O}$ ,  $^{20}\text{Ne}$ , etc:



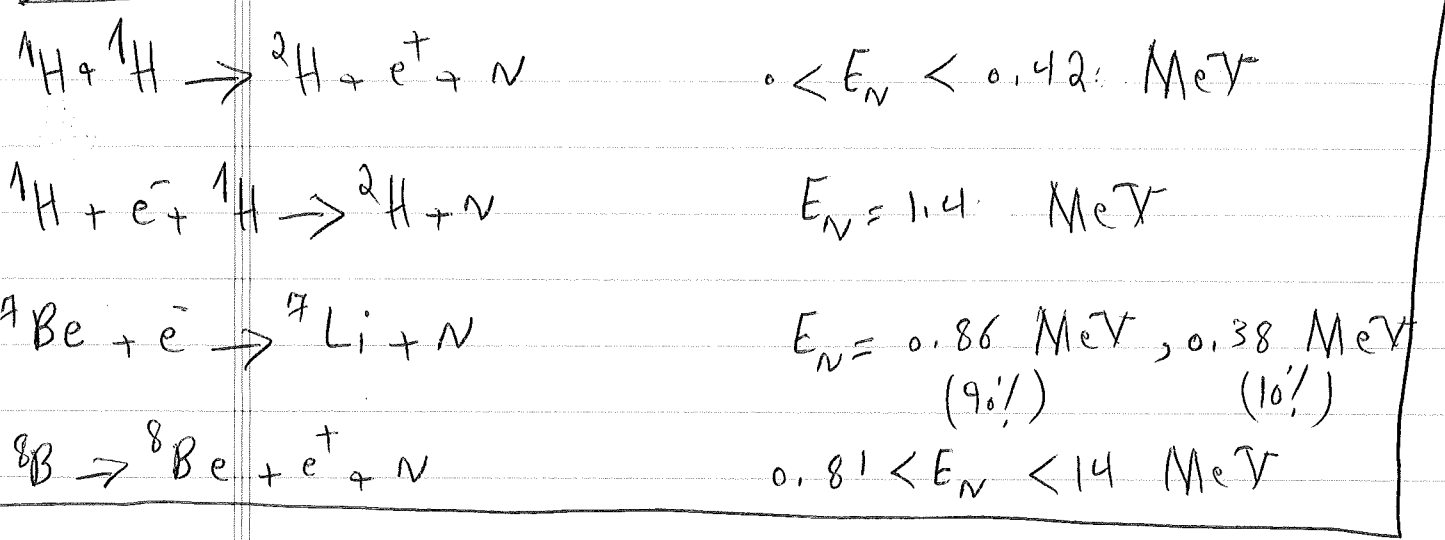
For a mixture consisting of  $^{13}\text{C}$  and  $^{16}\text{O}$  (after Helium burning) Carbon burning will set in if the temperature and density are high enough ( $T \geq 5 \times 10^8 \text{ K}$ ). We are not going to detailed discussion of Carbon burnings, etc. We just point out some of the complications that arise, namely resonances that can decay via many different channels and the very large number of reaction chains that have to be taken into account.

Neutrinos:

Neutrinos require special consideration because of their very tiny cross-sections. This implies that they can carry energy away from stellar matter.

Lets consider neutrinos produced in Hydrogen burning. The relevant processes in the pp-chain (which produces 98.5% of Sun energy) are:



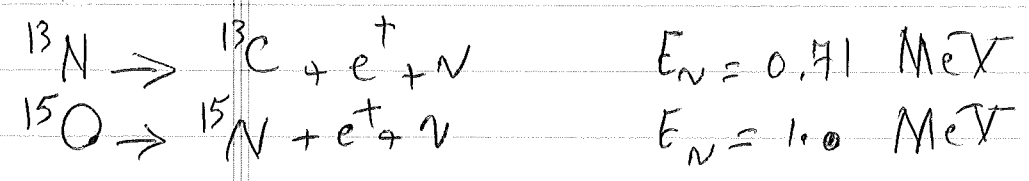


99% of neutrinos in Sun are produced in the  $\beta\beta$ -chain.

The reason for two lines in  ${}^7\text{Be}$  capture is that 10% of the time  ${}^7\text{Li}$  is produced in a metastable state, which then decays by emitting a photon.

We see that there are three neutrino lines and two continuous bands (one at low energy from  $\beta\beta$ -fusion, one at higher energies from Boron decay).

In addition, we have neutrino production in the CNO-cycle (which for Sun produces only 1.5% of <sup>its</sup> energy):



For scattering of neutrinos with energy  $E_\nu$  we roughly have:

$$\sigma_\nu \approx \left( \frac{E_\nu}{m_e c^2} \right)^2 10^{-44} \text{ cm}^2 \quad (m_e: \text{electron mass})$$

Thus neutrinos in the MeV range have  $\sigma_\nu \approx 10^{-44} \text{ cm}^2$ , which is  $\sim 10^{-20}$  smaller than the Thomson cross-section.

The corresponding mean free length of neutrinos in matter of density  $\rho$  is:

$$l_\nu = \frac{1}{n \sigma_\nu} \approx \frac{2 \times 10^{20} \text{ cm}}{\rho}$$

For  $\rho \sim 1 \text{ g cm}^{-3}$ , we find  $l_\nu \sim 100 \text{ pc}$ . Even for  $\rho = 10^6 \text{ g cm}^{-3}$ , one finds  $l_\nu \sim 3000 R_\odot$ . We conclude that neutrinos freely escape from Sun. As a rule of thumb, there exists one solar neutrino per  $\approx 13 \text{ MeV}$  of produced energy.

For solar luminosity ( $L_\odot \approx 4 \times 10^{33} \text{ erg s}^{-1}$ ), this results in  $\overset{\text{production of}}{\sim 2 \times 10^{38}}$  neutrinos per second, which yields in a flux  $\sim 10^{11}$  neutrinos per  $\text{cm}^2$  on Earth.

It is important to note that in extremely dense media neutrino mean free length will be very short. During a collapse in the final evolutionary stage of massive stars, the density can reach nuclear values  $\rho = 10^{14} \text{ g cm}^{-3}$  which gives rise to  $l_n \sim 20 \text{ km}$ . Neutrinos can then interact efficiently with the matter, just like as photons do inside Sun. Then it is necessary to consider a transport equation for neutrino energy.