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Nuclear Energy Production in Stars (Cont'd):

Because of the properties of thermonuclear reaction rates, different burnings are well separated by appreciable temperature differences. During a certain phase only very few reactions occur with non-negligi b/c rates. The important properties are: astrophysical "S" factor, correction factors, and energy release "Q". These can be found in the literature.

A whole network of all simultaneously occuring reactions must be calculated. Often enough, only the rate for the slowest of a chain of subsequent reaction suffices to be calculated, since it determines essentially the rate of the whole fusion process. However, the Q value contains all of the energy made available to the stellar matter by the process. Lets consider two of the major burning phases in stars.

Hydrogen Burning:

The net result of Hydrogen burning is the fusion of four ^1H into one ^4He and release of 26.73 MeV of energy in the process (which amounts to a mass defect of 0.71%). This is roughly 10 times the energy liberated in any other fusion process.

There are two main series of reactions known as proton-proton chain (pp-chain) and Carbon-Nitrogen-Oxygen cycle (CNO-cycle).

The pp-chain starts with ^2H formation and then ^3He formation:

$$^1H + {}^1H \rightarrow {}^2H + e^+ + \nu$$

$$^2H + {}^1H \rightarrow {}^3He + \gamma$$

The first reaction includes β^+ decay of a proton, which has a very small cross-section since the decay is governed by weak interactions.

Once ^3He is formed, the completion of ^4He can proceed via one of three alternative branches: pp1, pp2, and pp3. The reactions involved are shown below:

$$^1H + {}^1H \rightarrow {}^2H + e^+ + \nu$$

$$^2H + {}^1H \rightarrow {}^3He + \gamma$$

$$^3He + {}^3He \rightarrow {}^4He + 2\,{}^1H$$

(pp1)

$Q = 26.20$ MeV

$$^3He + {}^4He \rightarrow {}^7Be + \gamma$$

$$^7Be + e^- \rightarrow {}^7Li + \nu$$

$$^7Li + {}^1H \rightarrow {}^4He + {}^4He$$

(pp2)

$Q = 25.67$ MeV

$$^7Be + {}^1H \rightarrow {}^8B + \gamma$$

$$^8B \rightarrow {}^8Be + e^+ + \nu$$

$$^8Be \rightarrow {}^4He + {}^4He$$

(pp3)

$Q = 19.20$ MeV

The ^3He – ^4He reaction has a 14% larger reduced mass, a 4.6% larger J (see equation from previous lecture), and hence a slightly larger temperature sensitivity.

With increasing T, $pp2$ and $pp3$ dominate more and more over $pp1$ (say above $\simeq 10^7$ K) if ^4He is present with appreciable amounts. The relative importance will gradually shift toward $pp3$.

Initially $pp1$ is the main source of energy generation, say for $T \simeq 8\times10^6$ K). For gradually increasing T, and production of ^4He, the energy generation increases by a factor of $\simeq 2$ at $T \simeq 2\times10^7$ K. The $pp2$ then takes over as each ^1H-^1H reaction yields one ^4He (Compared to every second such reaction in the $pp1$ branch). The energy generation the decreases from $\simeq 2$ to $\simeq 1.5$ where $pp3$ has taken over (note that $pp3$ has a much smaller Q).

The temperature sensitivity of the pp-chain is the smallest of all fusions; $\nu \simeq 6$ at $T \simeq 5\times10^6$ K, which decreases to $\nu \simeq 3.5$ at $T \simeq 2\times10^7$ K..

Next we consider the CNO-cycle. It requires the presence of some isotopes of C, N, or O (as happens in Pop I stars). The sequence of reactions can be represented as follows:

$${}^{12}C + {}^{1}H \to {}^{13}N + \gamma$$

$${}^{13}N \to {}^{13}C + e^{+} + \nu$$

$${}^{13}C + {}^{1}H \to {}^{14}N + \gamma$$

$${}^{14}N + {}^{1}H \to {}^{15}O + \gamma$$

$${}^{15}O \to {}^{15}N + e^{+} + \nu$$

$${}^{15}N + {}^{1}H \to {}^{12}C + {}^{4}He$$

Main Cycle

$Q = 24.97$ MeV

$$\to {}^{16}O + \gamma$$

$${}^{16}O + {}^{1}H \to {}^{17}F + \gamma$$

$${}^{17}F \to {}^{17}O + e^{+} + \nu$$

$${}^{17}O + {}^{1}H \to {}^{14}N + {}^{4}He$$

Secondary Cycle

The secondary cycle is $\sim 10^{4}$ times less probable than the main cycle. Most stars change slowly enough that for sufficiently high temperatures (say $T \gtrsim 1.5\times10^{7}$ K) all the nuclei in the cycle reach their equilibrium abundance. Then it is sufficient to calculate only the slowest reaction, which is $^{14}N + ^{1}H$ that essentially controls the cycle. This slowest reaction acts like a bottleneck where the nuclei involved are congested in their flow through the cycle.

The temperature sensitivity ν is much higher than that in the pp-chain. For $T \simeq 10^{7}$ K (5×10^{7} K) we have $\nu \simeq 23$ (13). In consequence, the pp-chain dominates at low temperatures ($T < 1.5\times10^{7}$ K), while it can be neglected at higher temperatures.

Helium Burning:

The reactions consist of the gradual fusion of several ^{4}He into

^{12}C, ^{16}O, etc. This requires temperatures $\gtrsim 10^8$ K (because of the larger Coulomb barrier).

The first and key reaction is the triple α reaction, which is performed in two steps:

$^4He + {}^4He \rightleftarrows {}^8Be$

$Q = 7.28$ MeV

$^8Be + {}^4He \rightarrow {}^{12}C + \gamma$

The ^8Be decays back into two α particles with a lifetime of few times 10^{-16} s. This is $\sim 10^5$ larger than the duration of a normal scattering process, hence ^8Be can participate in the second reaction in above.

Once a sufficient ^{12}C abundance has been built, further α capture can occur to form ^{16}O, ^{20}Ne, etc:

$^{12}C + {}^4He \rightarrow {}^{16}O + \gamma$

$^{16}O + {}^4He \rightarrow {}^{20}Ne + \gamma$

⋮

For a mixture consisting of ^{12}C and ^{16}O (after Helium burning) Carbon burning will set in if the temperature and density are high enough ($T \gtrsim 5\times10^{8}$ K). We are not going to detailed discussion of Carbon burning, etc. We just point out some of the complications that arise, namely resonances that can decay via many different channels and the very large number of reaction chains that have to be taken into account.

Neutrinos:

Neutrinos require special consideration because of their very tiny cross-seetions. This implies that they can carry energy away from stellar matter.

Lets consider neutrinos produced in Hydrogen burning. The relevant processes in the pp-chain (which produces 98.5% of Sun energy) are:

$^1H + {}^1H \rightarrow {}^2H + e^+ + \nu$	$0 < E_\nu < 0.42$ MeV
$^1H + e^- + {}^1H \rightarrow {}^2H + \nu$	$E_\nu = 1.4$ MeV
$^7Be + e^- \rightarrow {}^7Li + \nu$	$E_\nu = 0.86$ MeV, 0.38 MeV (90%) (10%)
$^8B \rightarrow {}^8Be + e^+ + \nu$	$0.81 < E_\nu < 14$ MeV

99% of neutrinos in Sun are produced in the pp-chain.

The reason for two lines in ^7Be capture is that 10% of the time ^7Li is produced in a metastable state, which then decays by emitting a photon.

We see that there are three neutrino lines and two continuous bands (one at low energy from pp-fusion, one at higher energies from Boron decay).

In addition, we have neutrino production in the CNO-cycle (which for Sun produces only 1.5% of its energy):

$^{13}N \rightarrow {}^{13}C + e^+ + \nu$ $E_\nu = 0.71$ MeV

$^{15}O \rightarrow {}^{15}N + e^+ + \nu$ $E_\nu = 1.0$ MeV

For scattering of neutrinos with energy E_ν we roughly have:

$$\boxed{\sigma_\nu \approx \left(\frac{E_\nu}{m_e c^2}\right) 10^{-44} \text{ cm}^2 \quad (m_e:\text{ electron mass})}$$

Thus neutrinos in the MeV range have $\sigma_\nu \approx 10^{-44}$ cm^2, which is $\sim 10^{-20}$ smaller than the Thomson cross-section.

The corresponding mean free length of neutrinos in matter of density ρ is:

$$\boxed{\ell_\nu = \frac{1}{n\sigma_\nu} \approx \frac{2\times 10^{20}\text{ cm}}{\rho}}$$

For $\rho \sim 1$ g cm^{-3}, we find $\ell_\nu \sim 100$ pc. Even for $\rho = 10^6$ g cm^{-3}, one finds $\ell_\nu \sim 3000\, R_\odot$. We conclude that neutrinos freely escape from Sun. As a rule of thumb, there exists one solar neutrino per ≈ 13 MeV of produced energy. For Solar luminosity ($L_\odot \approx 4\times 10^{33}$ erg s^{-1}), this results in production of 2×10^{38} neutrinos per second, which yields in a flux $\sim 10^{11}$ neutrinos per cm^2 on Earth.

It is important to note that in extremely dense media neutrino mean free length will be very short. During a collapse in the final evolutionary stage of massive stars, the density can reach nuclear values $\rho \simeq 10^{14}\ g\ cm^{-3}$, which gives rise to $l_\nu \sim 20$ km. Neutrinos can then interact efficiently with the matter, just like as photons do inside Sun. Then it is necessary to consider a transport equation for neutrino energy.